

Chapter 30 Homework Solutions

Solutions for the following Self Tutoring and Skill Builder problems are available through Mastering Physics: Energy in Capacitors and Electric Fields, Potential of a Charged Disk, and Potential of Two Charged Spheres. To access these solutions, click on the “View Solution” link on each problem view window.

30.4. Model: The electric field is the negative of the derivative of the potential function.

Solve: From Equation 30.8, the component of the electric field in the s -direction is $E_s = -dV/ds$. For the given potential,

$$\frac{dV}{dx} = \frac{d}{dx}(100x^2 \text{ V}) = 200x \frac{\text{V}}{\text{m}} \Rightarrow E_x = -200x \text{ V/m}$$

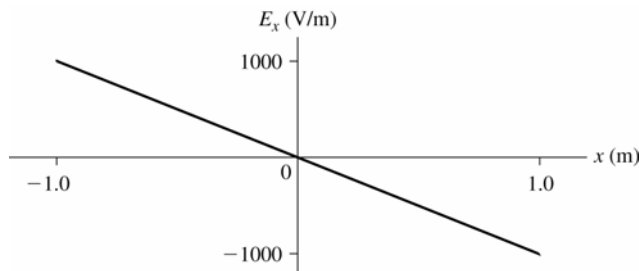
At $x = 0 \text{ m}$, $E_x = 0 \text{ V/m}$, and at $x = 1 \text{ m}$, $E_x = -200(1) \text{ V/m} = -200 \text{ V/m}$.

Assess: The potential increases with x , so the electric field must point in the $-x$ direction.

30.32. Solve: The energy density is $u_E = (5.0 \times 10^{-12} \text{ J}) / (0.02 \text{ m} \times 0.02 \text{ m} \times 0.02 \text{ m}) = 6.25 \times 10^{-7} \text{ J/m}^3$.

$$u_E = \frac{1}{2} \epsilon_0 E^2 \Rightarrow E = \sqrt{\frac{2u_E}{\epsilon_0}} = \sqrt{\frac{2(6.25 \times 10^{-7} \text{ J/m}^3)}{8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2}} = 1190 \text{ V/m}$$

30.35. Solve: (a)



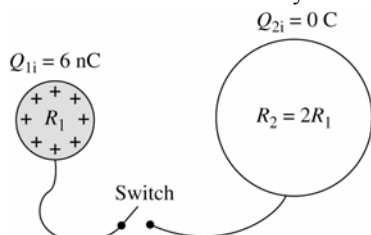
(b) Equation 30.3 gives the potential difference between two points in space:

$$\begin{aligned} \Delta V &= -\int_{x_i}^{x_f} E_x dx = -\int_{-0.20 \text{ m}}^{0.30 \text{ m}} (-1000x \text{ V/m}) dx \\ &= +\left[1000 \frac{x^2}{2} \right]_{-0.20 \text{ m}}^{0.30 \text{ m}} \text{ V} = +\frac{1000}{2} [(0.30 \text{ m})^2 - (-0.20 \text{ m})^2] \text{ V} = +65 \text{ V} \end{aligned}$$

Assess: E is positive for negative x but negative for positive x , so the potential difference depends on the square of the positions.

30.47. Model: Conductors connected by a conducting wire are at the same potential.

Visualize:



Charge flows from sphere 1 to sphere 2, when the wire is connected. Once connected, the entire system is one large conductor in electrostatic equilibrium. Consequently, the entire system must be an equipotential. That is, the two spheres must have equal potentials $V_1 = V_2$ after being connected by the wire.

Solve: The potential of a sphere is $V = Q/4\pi\epsilon_0 R$, so the final charges Q_{1f} and Q_{2f} on the two spheres must be such that

$$\frac{Q_{1f}}{4\pi\epsilon_0 R_1} = \frac{Q_{2f}}{4\pi\epsilon_0 R_2} = \frac{Q_{2f}}{4\pi\epsilon_0 (2R_1)} \Rightarrow Q_{1f} = \frac{1}{2} Q_{2f}$$

where we used the fact that $R_2 = 2R_1$. It is also true that charge must be conserved, so

$$Q_{1f} + Q_{2f} = Q_{1i} + Q_{2i} = 6 \text{ nC}$$

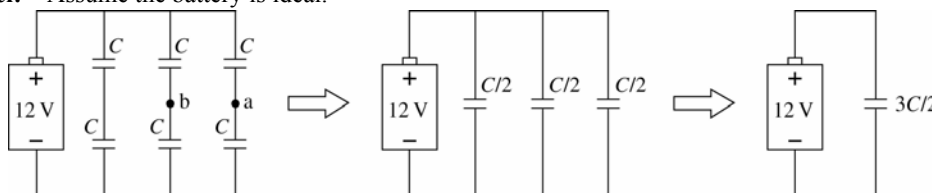
Using $Q_{2f} = 2Q_{1f}$, we find

$$Q_{1f} + 2Q_{1f} = 3Q_{1f} = 6 \text{ nC}$$

from which we finally get $Q_{1f} = 2 \text{ nC}$ and $Q_{2f} = 4 \text{ nC}$.

30.68. Model: Assume the battery is ideal.

Visualize:



The circuit in Figure P30.68 has been redrawn to show that the six capacitors are arranged in three parallel combinations, each combination being a series combination of two capacitors.

Solve: (a) The equivalent capacitance of the two capacitors in series is $\frac{1}{2}C$. The equivalent capacitance of the six capacitors is $\frac{3}{2}C$.

(b) As points a and b are midpoints of identical capacitors, $V_a = V_b = 6.0 \text{ V}$. Therefore, the potential difference between points a and b is zero.

30.74. Solve: (a) The capacitance of the parallel-plate capacitor is

$$C_1 = \frac{A\epsilon_0}{d_1} = \frac{(0.1 \text{ m} \times 0.1 \text{ m})(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)}{1.0 \times 10^{-3} \text{ m}} = 8.85 \times 10^{-11} \text{ F}$$

The electric potential energy stored in the capacitor is

$$U_1 = \frac{1}{2} \frac{Q^2}{C_1} = \frac{1}{2} \frac{(10 \times 10^{-9} \text{ C})^2}{8.85 \times 10^{-11} \text{ F}} = 5.65 \times 10^{-7} \text{ J}$$

(b) There is no change in the charge. The energy change is due to the change in the capacitance. The new capacitance is

$$C_2 = \frac{\epsilon_0 A}{d_2} = \frac{\epsilon_0 A}{2d_1} = \frac{C_1}{2}$$

The amount of energy stored is $U_2 = 11.3 \times 10^{-7} \text{ J}$.

(c) Work was done on the capacitor by the agent pulling the plates apart, thereby adding energy into the system.